Solving the Joint Order Batching and Picker Routing Problem for Large Instances

Final Presentation

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4th April 2019

Introduction (About Us I)

Wei Hao Khoong

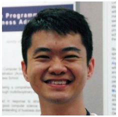
National University of Singapore

A final year undergraduate majoring in Applied Mathematics, with a specialization in operations research and financial mathematics.

Dr Melvin Zhang[1]

Co-founder + CTO Cosmiqo International Pte Ltd

An avid programmer who enjoys designing and implementing novel algorithms.



[1] Retrieved from: `https://cosmiqo.com/index.html`

Outline

1. The Project (Recap)
    - Objectives

2. Literature Review & The Working Model
    - Preliminaries
    - The Order Batching Problem
    - The Picker Routing Problem
    - The Joint Order Batching and Picker Routing Problem

3. Solving the JOBPRP for Large Instances
    - Heuristics
    - Implementation
    - Experiments & Results
    - Conclusion & Future Work

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The Project

Objectives

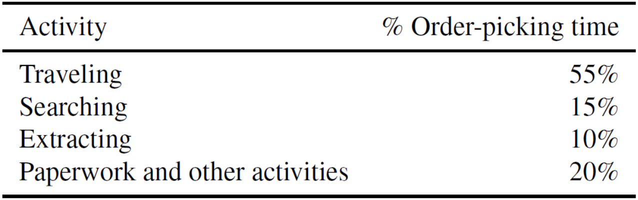
- Investigate methods to optimize order picking
  - by batching several orders together
  - by planning a good route to **minimize the distance** required to pick up all the items
- Proposed method should scale up to typical warehouse sizes

What is order picking?

- Retrieving products in an order from storage blocks in a warehouse
- Accounts for as much as 60% of all labor activities[2]

[2] Noud Gademann & Van De SteeF Velde, 2005. Order batching to minimize total travel time in a parallel-aisle warehouse.

What is order picking?



Activity	% Order-picking time
Traveling	55%
Searching	15%
Extracting	10%
Paperwork and other activities	20%

Breakdown of order-picking costs[3]

[3] John J Bartholdi and Steven T Hackman, 2017. Warehouse & distribution science release 0.98.

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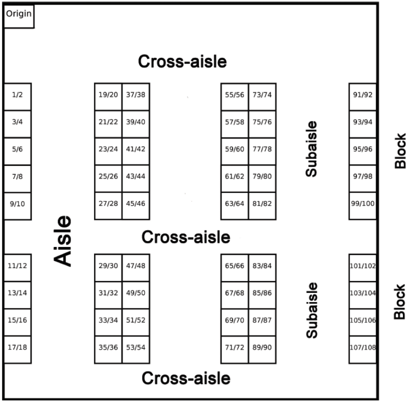
Important Definitions I

The Warehouse

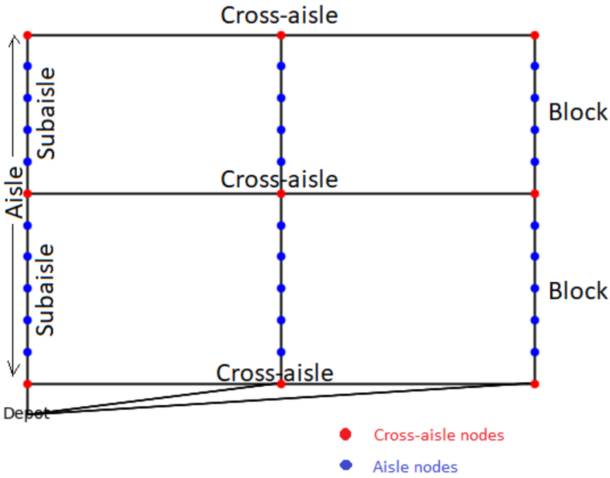


A warehouse structure[4]

[4] Cristiano Arbex Valle, John E. Beasley, and Alexandre Salles da Cunha, 2017. Optimally solving the joint order batching and picker routing problem.

Important Definitions II

The Warehouse



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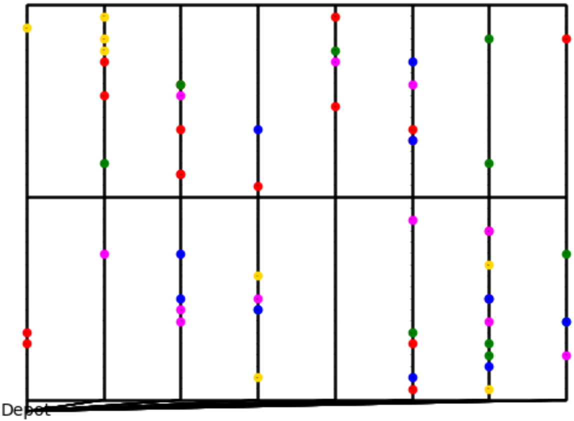
Problem Description

The Order Batching Problem - Input

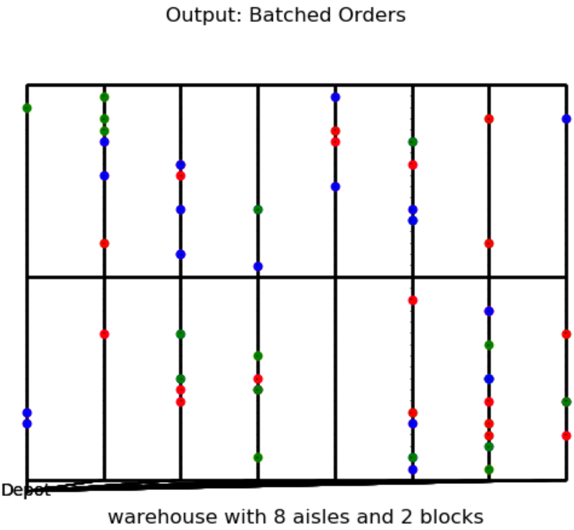
Input: Orders before batching



warehouse with 8 aisles and 2 blocks

Problem Description

The Order Batching Problem - Output



Problem Description

The Order Batching Problem

- Group orders received into a set of batches
- Time required to collect all the orders is minimized
- Problem is $\mathcal{NP}$-hard[5]

[5] Noud Gademann & Van De SteeF Velde, 2005. Order batching to minimize total travel time in a parallel-aisle warehouse.

Literature Review

The Order Batching Problem - Exact Solving Methods

- Branch-and-price algorithm to solve modest sized warehouse instances to optimality[6]
- Iterated descent approximation algorithm was suggested for larger warehouse instances
- Joint Order Batching & Picker Routing Problem (JOBPRP) was formulated and solved with a branch-and-cut algorithm[7]

[6] Noud Gademann & Van De SteeF Velde, 2005. Order batching to minimize total travel time in a parallel-aisle warehouse.

[7] Cristiano Arbex Valle, John E. Beasley, and Alexandre Salles da Cunha, 2017. Optimally solving the joint order batching and picker routing problem.

Literature Review

The Order Batching Problem - Heuristic Solving Methods

- Cluster analysis of orders[8]
- Seed-order selection rules[9]
- Time Savings[10]

[8] H. Hwang & D. G. Kim, 2005. Order-batching heuristics based on cluster analysis in a lowlevel picker-to-part warehousing system.

[9] Ying-Chin Ho, Teng-Sheng Su & Zhi-Bin Shi, 2008. Order-batching methods for an order-picking warehouse with two cross aisles.

[10] M. B. M. De Koster , E.S. Van der Poort & M. Wolters, 1999. Efficient orderbatching methods in warehouses.

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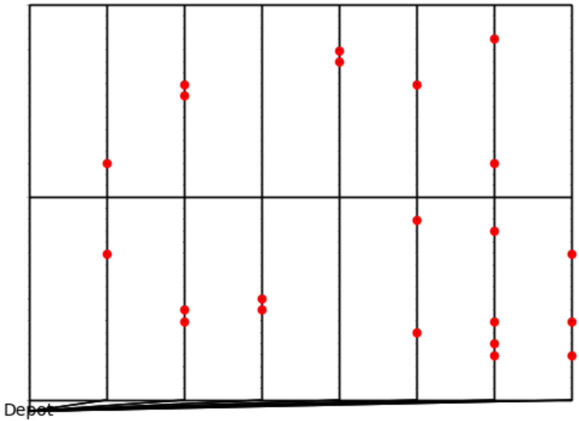
Problem Description

The Picker Routing Problem - Input

Input 1: Pick Locations of Picks In Batch 1

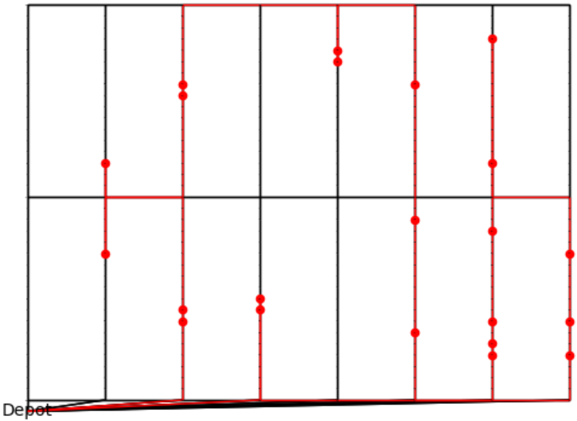


warehouse with 8 aisles and 2 blocks

Problem Description

The Picker Routing Problem - Output

Output 1: Route Traversed By Picker To Pick All Items In Batch 1



warehouse with 8 aisles and 2 blocks

Problem Description

The Picker Routing Problem

- Each picker is assigned a batched order
- Determine an optimal path to pick the items in each batch
- Problem is equivalent to the Travelling Salesman Problem (TSP)
- Problem is also $\mathcal{NP}$-hard[11]

[11] Wolsey L. A., 1998. Integer Programming, Wiley New York.

Literature Review

The Picker Routing Problem - Exact Solving Methods

- Concorde[12] TSP Solver
- Solved polynomially via a dynamic programming approach[13]

[12] David L. Applegate, Robert E. Bixby, Vasek Chvatal, and William J, 2007. Cook. The Traveling Salesman Problem: A Computational Study (Princeton Series in Applied Mathematics).

[13] Cristiano Arbex Valle, John E. Beasley, and Alexandre Salles da Cunha, 2017. Optimally solving the joint order batching and picker routing problem.

Literature Review

The Picker Routing Problem - Heuristic Solving Methods

- S-shape, Largest Gap, Combined+ routing heuristics[14]

[14] Kees Jan Roodbergen and RenÉde Koster, 2001. Routing methods for warehouses with multiple cross aisles.

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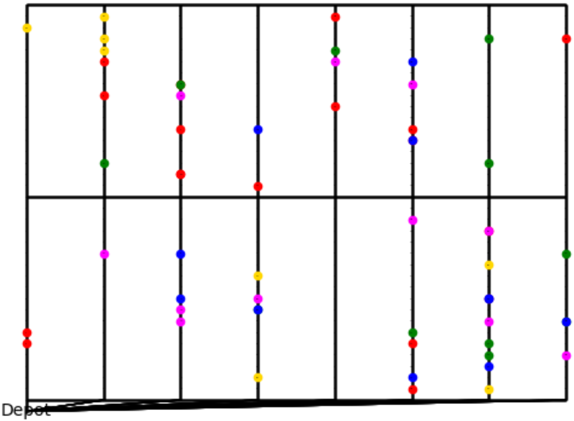
The Problem

JOBPRP - Input

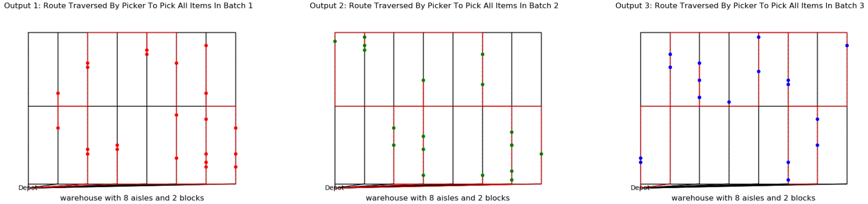
Input: Orders before batching



warehouse with 8 aisles and 2 blocks

The Problem

JOBPRP - Outputs



Output 1: Route Traversed By Picker To Pick All Items In Batch 1

Output 2: Route Traversed By Picker To Pick All Items In Batch 2

Output 3: Route Traversed By Picker To Pick All Items In Batch 3

Depot
warehouse with 8 aisles and 2 blocks
Depot
warehouse with 8 aisles and 2 blocks
Depot
warehouse with 8 aisles and 2 blocks

Outputs - 3 routed batched orders

Cost Function & Main Constraints[15]

- **Cost function**: To minimize total distance traversed by all the pickers
- **Capacity constraint**: To ensure picker capacity is not exceeded
- **Order assignment constraint**: To ensure each order is picked by only one picker

[15] Cristiano Arbex Valle, John E. Beasley, and Alexandre Salles da Cunha, 2017. Optimally solving the joint order batching and picker routing problem.

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Heuristics

Overview and Trade-offs

- The JOBPRP is $\mathcal{NP}$-hard[16]
- We use heuristics to achieve "good" enough feasible solutions in a short time

[16] Cristiano Arbex Valle, John E. Beasley, and Alexandre Salles da Cunha, 2017. Optimally solving the joint order batching and picker routing problem.

Heuristics Used

Overview

Picker Routing Heuristics[17]**:** S-shape, Largest Gap

Batching Heuristic(s): Time Savings[18]

[17] Kees Jan Roodbergen & RenÉde Koster, 2001. Routing methods for warehouses with multiple cross aisles.

[18] M. B. M. De Koster , E.S. Van der Poort & M. Wolters, 1999. Efficient orderbatching methods in warehouses.

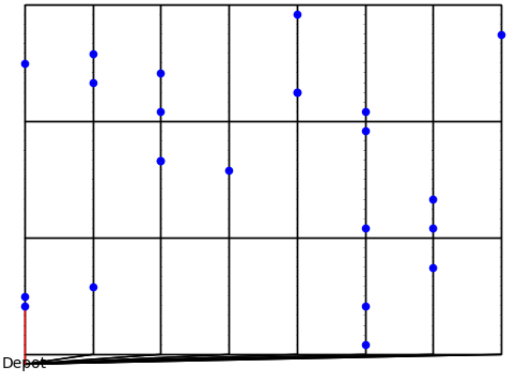
Heuristics Used I

S-shape

Here's an example to illustrate how the S-shape heuristic works:

Heuristics Used II

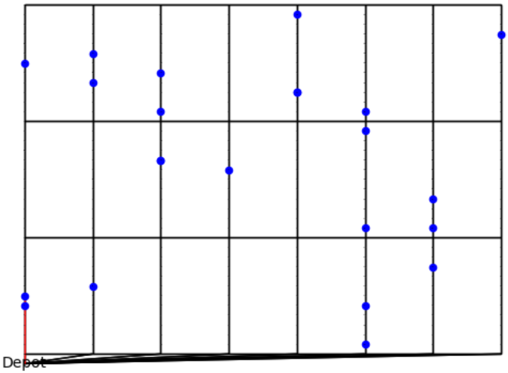
S-shape



Total Distance = 336.6m

Heuristics Used III

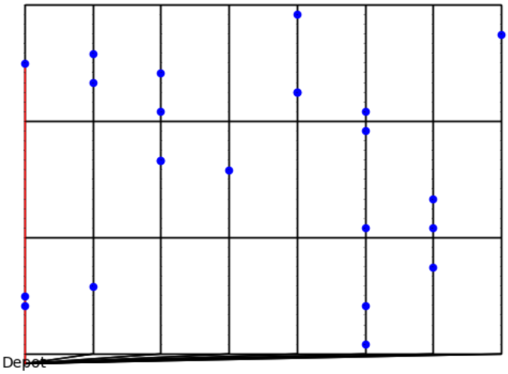
S-shape



Total Distance = 336.6m

Heuristics Used IV

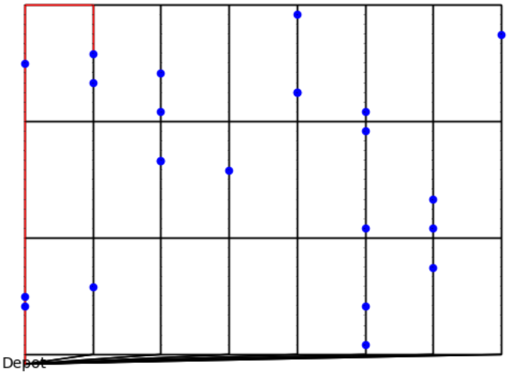
S-shape



Total Distance = 336.6m

Heuristics Used V

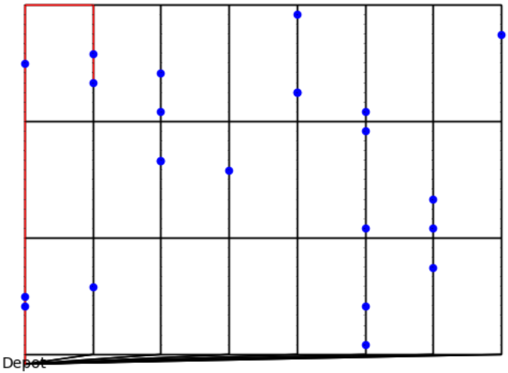
S-shape



Total Distance = 336.6m

Heuristics Used VI

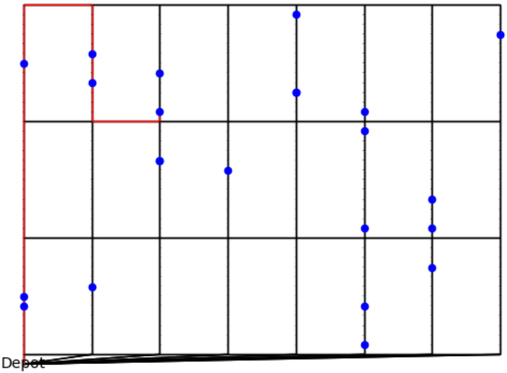
S-shape



Total Distance = 336.6m

Heuristics Used VII

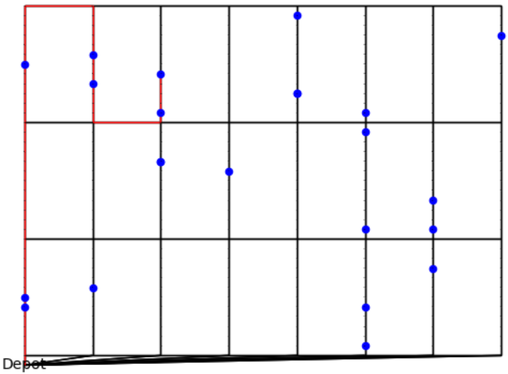
S-shape



Total Distance = 336.6m

Heuristics Used VIII

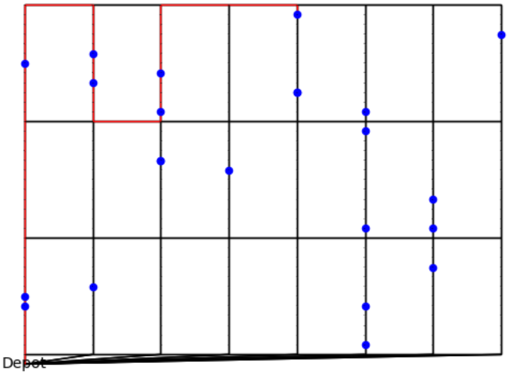
S-shape



Total Distance = 336.6m

Heuristics Used IX

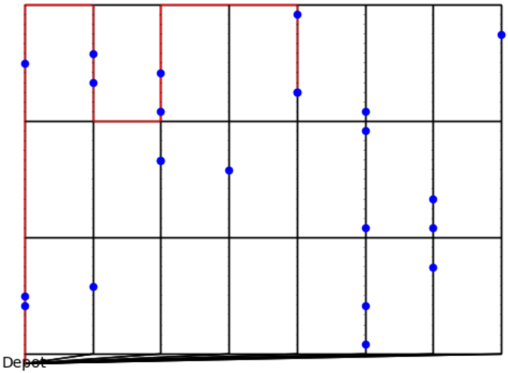
S-shape



Total Distance = 336.6m

Heuristics Used X

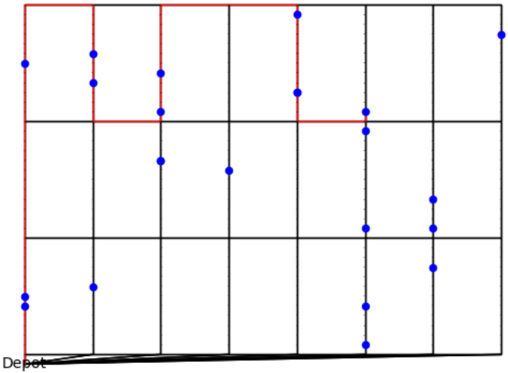
S-shape



Total Distance = 336.6m

Heuristics Used XI

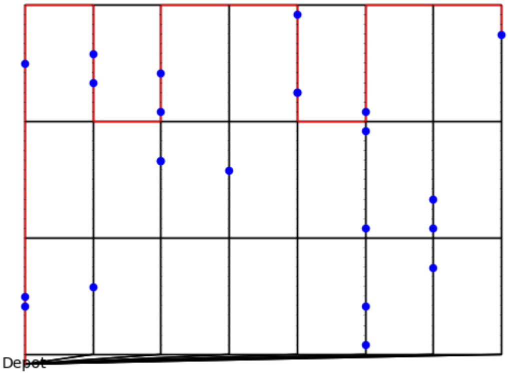
S-shape



Total Distance = 336.6m

Heuristics Used XII

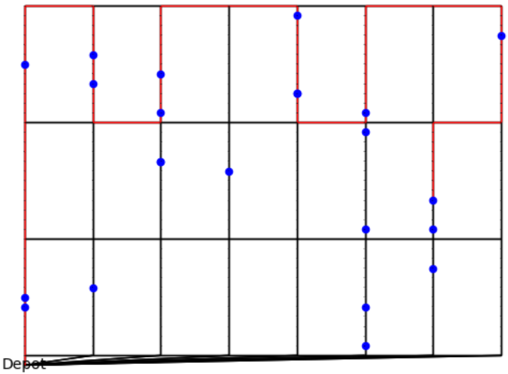
S-shape



Total Distance = 336.6m

Heuristics Used XIII

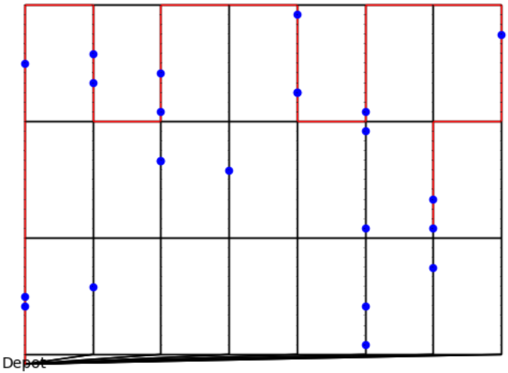
S-shape



Total Distance = 336.6m

Heuristics Used XIV

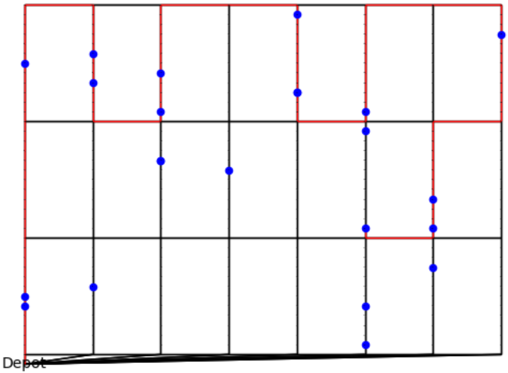
S-shape



Total Distance = 336.6m

Heuristics Used XV

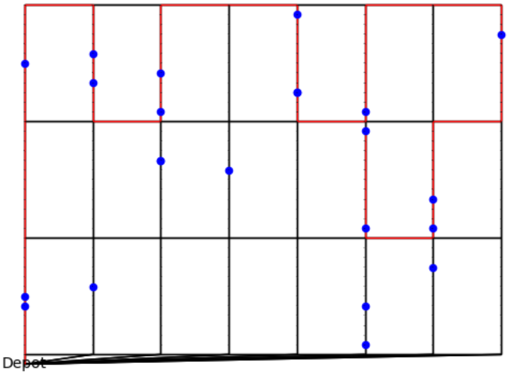
S-shape



Total Distance = 336.6m

Heuristics Used XVI

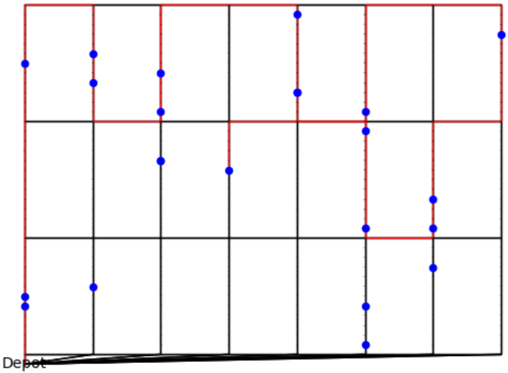
S-shape



Total Distance = 336.6m

Heuristics Used XVII

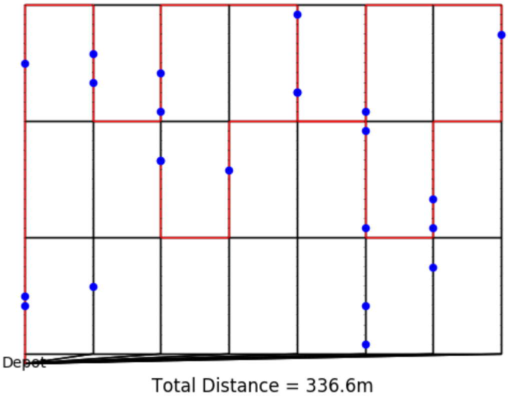
S-shape



Total Distance = 336.6m

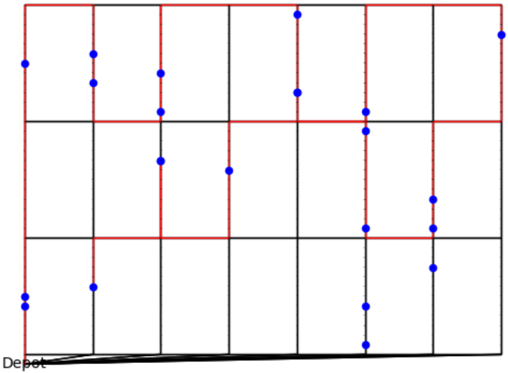
Heuristics Used XVIII

S-shape



Heuristics Used XIX

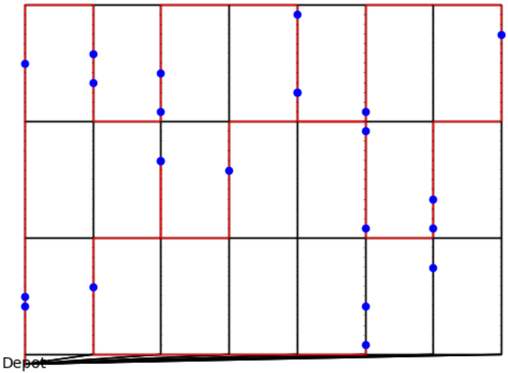
S-shape



Total Distance = 336.6m

Heuristics Used XX

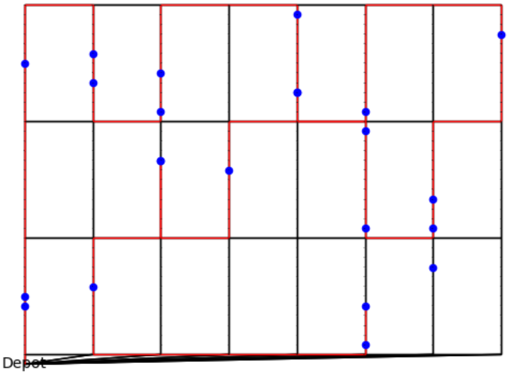
S-shape



Total Distance = 336.6m

Heuristics Used XXI

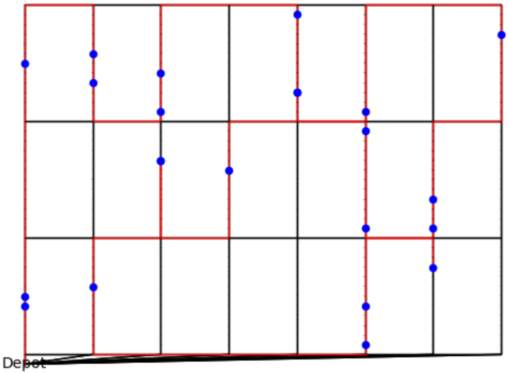
S-shape



Total Distance = 336.6m

Heuristics Used XXII

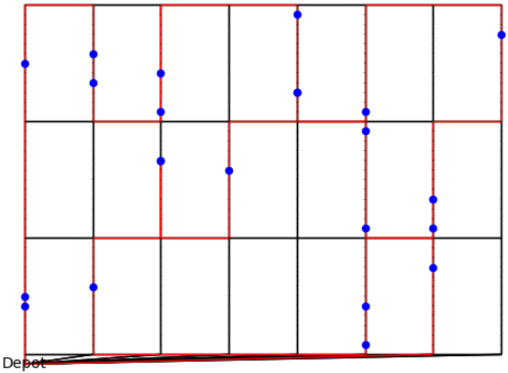
S-shape



Total Distance = 336.6m

Heuristics Used XXIII

S-shape



Total Distance = 336.6m

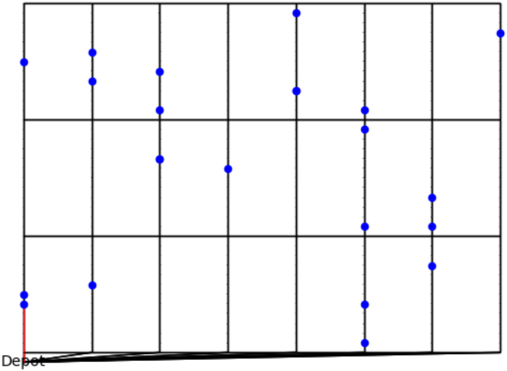
Heuristics Used I

Largest Gap

Here's an example to illustrate how the Largest Gap heuristic works:

Heuristics Used II

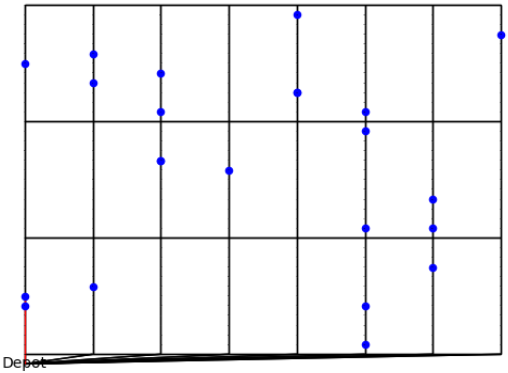
Largest Gap



Total Distance = 380.62m

Heuristics Used III

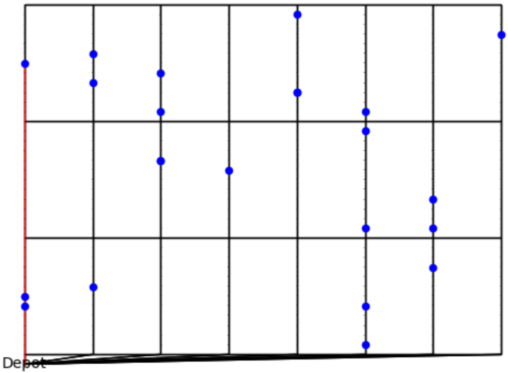
Largest Gap



Total Distance = 380.62m

Heuristics Used IV

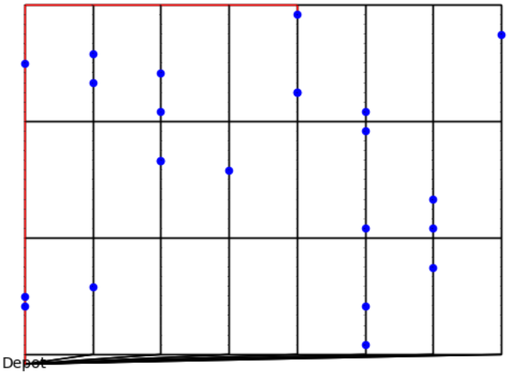
Largest Gap



Total Distance = 380.62m

Heuristics Used V

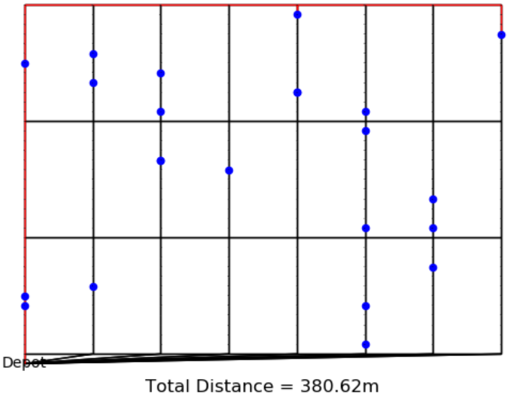
Largest Gap



Total Distance = 380.62m

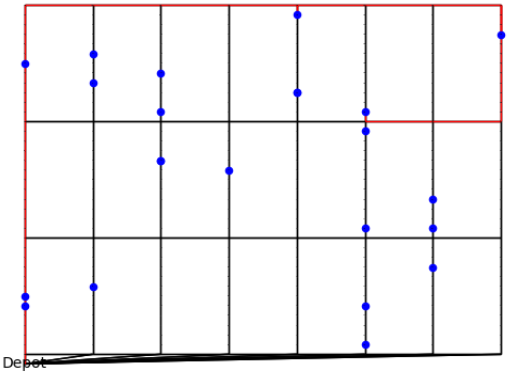
Heuristics Used VI

Largest Gap



Heuristics Used VII

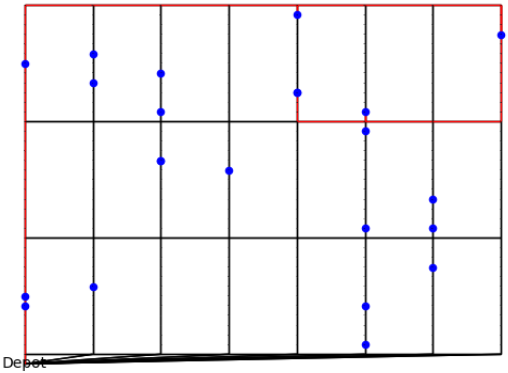
Largest Gap



Total Distance = 380.62m

Heuristics Used VIII

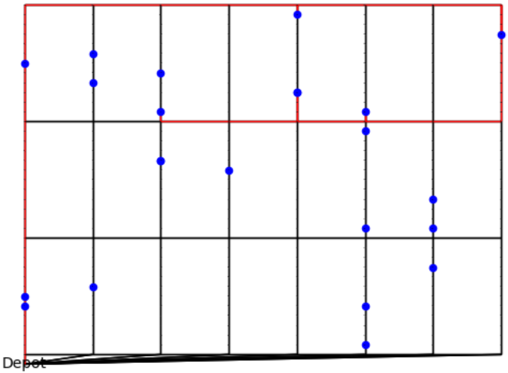
Largest Gap



Total Distance = 380.62m

Heuristics Used IX

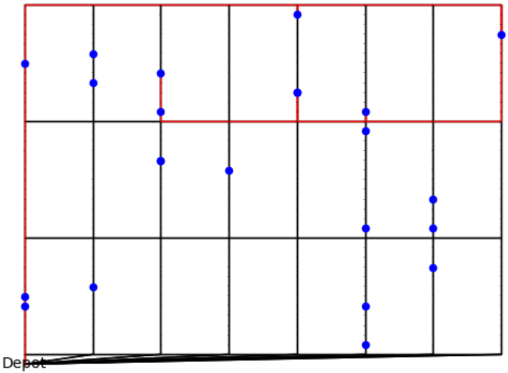
Largest Gap



Total Distance = 380.62m

Heuristics Used X

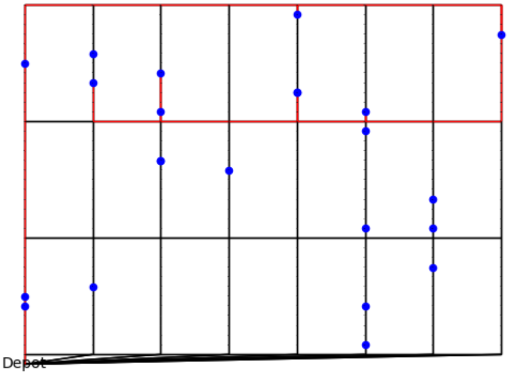
Largest Gap



Total Distance = 380.62m

Heuristics Used XI

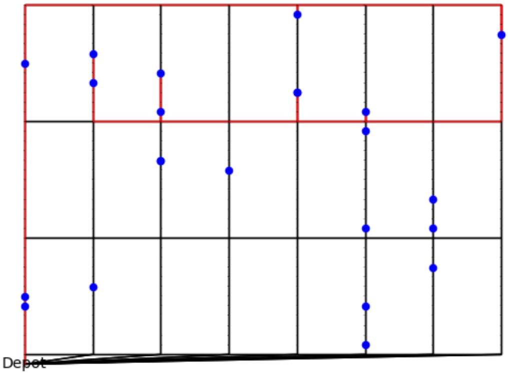
Largest Gap



Total Distance = 380.62m

Heuristics Used XII

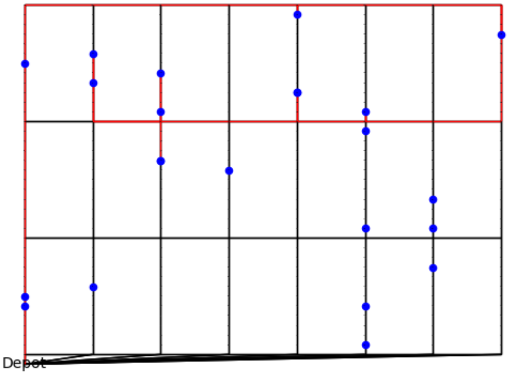
Largest Gap



Total Distance = 380.62m

Heuristics Used XIII

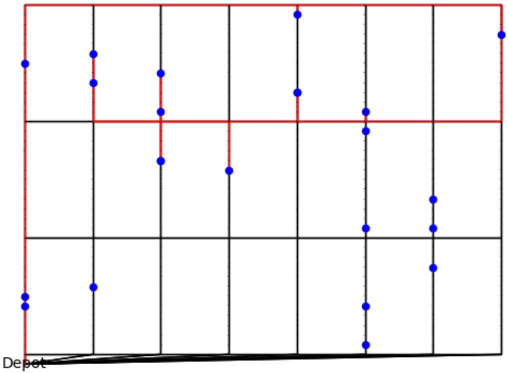
Largest Gap



Total Distance = 380.62m

Heuristics Used XIV

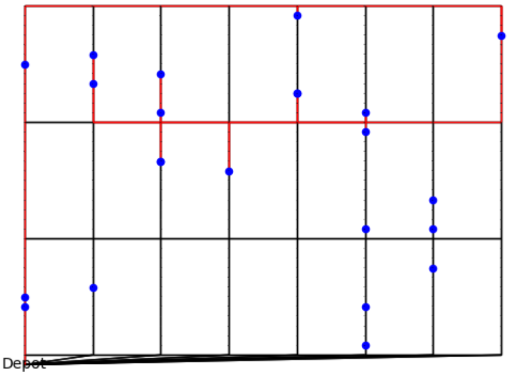
Largest Gap



Total Distance = 380.62m

Heuristics Used XV

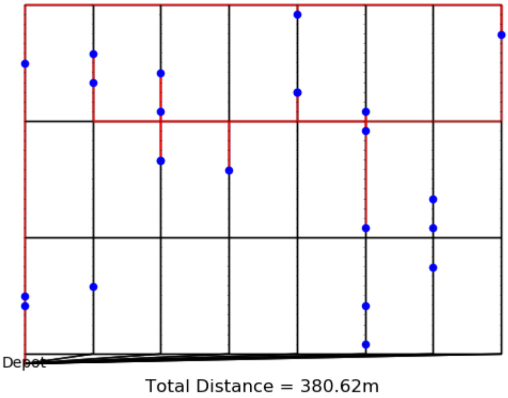
Largest Gap



Total Distance = 380.62m

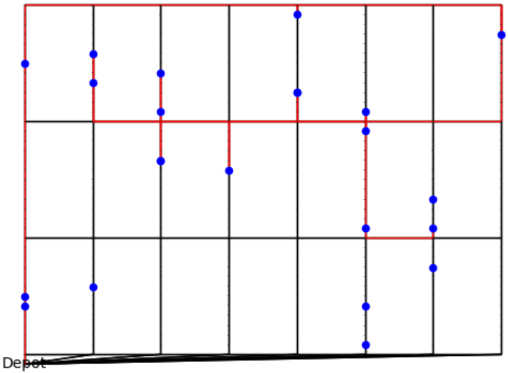
Heuristics Used XVI

Largest Gap



Heuristics Used XVII

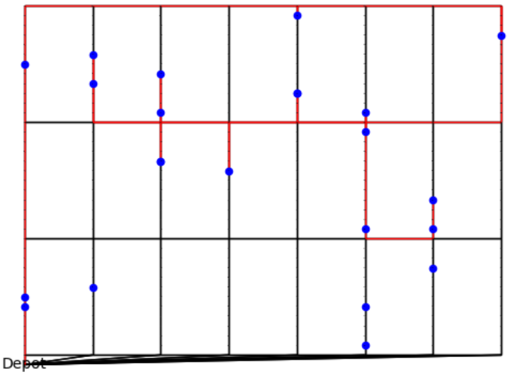
Largest Gap



Total Distance = 380.62m

Heuristics Used XVIII

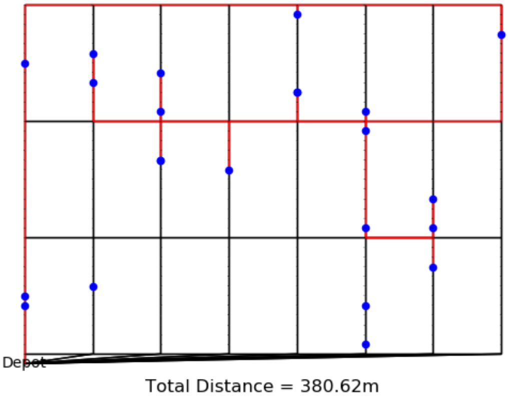
Largest Gap



Total Distance = 380.62m

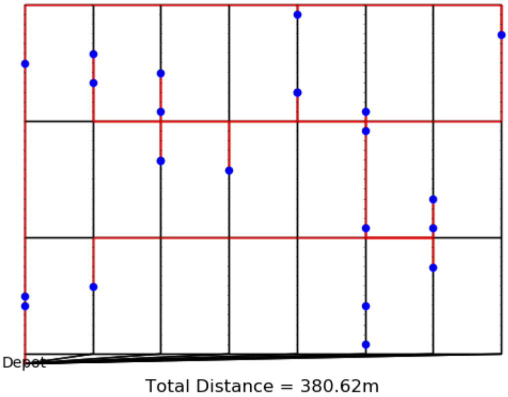
Heuristics Used XIX

Largest Gap



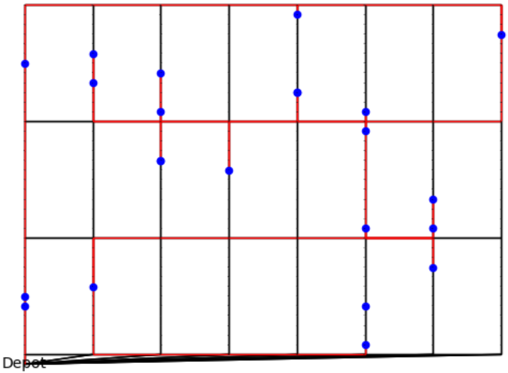
Heuristics Used XX

Largest Gap



Heuristics Used XXI

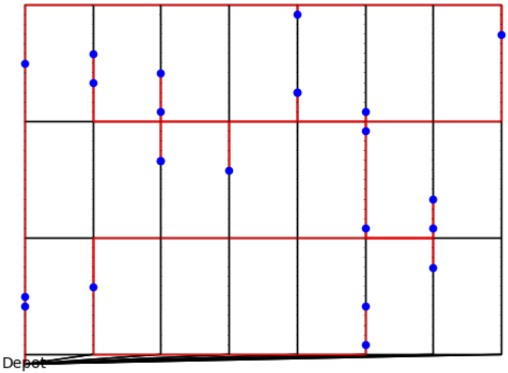
Largest Gap



Total Distance = 380.62m

Heuristics Used XXII

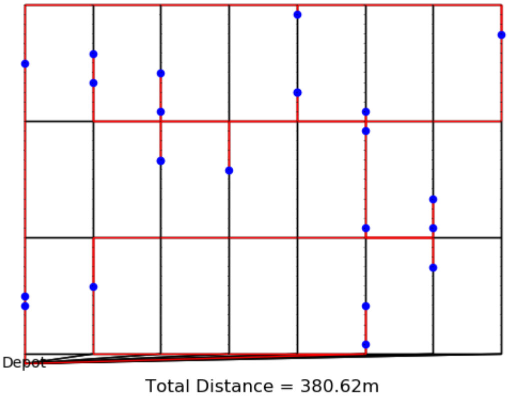
Largest Gap



Total Distance = 380.62m

Heuristics Used XXIII

Largest Gap



Heuristics Used I

Time Savings - Preliminaries

Picker Routing Heuristics: S-shape, Largest Gap

Batching Heuristic(s): Time Savings

Heuristics Used I

Time Savings - Purpose

- Merge orders in a greedy manner according to savings saved[19]
- Fewer batches of orders results in less distance travelled

Heuristics Used II

Time Savings - Purpose

Definition 1 (Savings)

For any 2 orders o_i and o_j, we have for a savings router sr,

$$\text{savings}_{ij} = sr(o_i) + sr(o_j) - sr(o_i \cup o_j)$$

where $sr(o_i)$ is the distance travelled to pick all items in order o_i.

[19] Clarke, G. and Wright, J.R. (1964) Scheduling of Vehicle Routing Problem from a Central Depot to a Number of Delivery Points.

Heuristics Used

Time Savings - An Example

Order i	1	2	3	4	5	6	7
1	-						
2	X	-					
3	74	X	-				
4	64	58	74	-			
5	68	X	94	54	-		
6	X	X	X	78	68	-	
7	-2	10	10	-4	10	-10	-

An example of a savings matrix[20]

Order i	Weight w_i
1	4
2	6
3	4
4	2
5	3
6	5
7	1

Weights of the orders

[20] M. B. M. De Koster , E.S. Van der Poort & M. Wolters, 1999. Efficient orderbatching methods in warehouses.

Heuristics Used I

Time Savings - An Example

Order i	1	2	3	4	5	6	7
1	-						
2	X	-					
3	74	X	-				
4	64	58	74	-			
5	68	X	94	54	-		
6	X	X	X	78	68	-	
7	-2	10	10	-4	10	-10	-

Order i	Weight w_i
1	4
2	6
3	4
4	2
5	3
6	5
7	1

Picker capacity $= 8$, Batches $= \{ \}$

1. Select pair with highest savings

$$\max \text{savings}_{ij} \rightarrow \{3, 5\}$$

$$\text{Batches} \rightarrow \text{Batches} + \{3, 5\}$$

Heuristics Used II

Time Savings - An Example

Order i	1	2	3	4	5	6	7
1	-						
2	X	-					
3	74	X	-				
4	64	58	74	-			
5	68	X	94	54	-		
6	X	X	X	78	68	-	
7	-2	10	10	-4	10	-10	-

Order i	Weight w_i
1	4
2	6
3	4
4	2
5	3
6	5
7	1

$$\text{Picker capacity} = 8, \text{ Batches} = \Big\{\{3,5\}\Big\}$$

2. Select next largest savings $\{4,6\}$. Since $w_4 + w_6 = 7 < 8$; Batches $\rightarrow$ Batches $+ \{4,6\}$

Heuristics Used III

Time Savings - An Example

Order i	1	2	3	4	5	6	7
1	-						
2	X	-					
3	74	X	-				
4	64	58	74	-			
5	68	X	94	54	-		
6	X	X	X	78	68	-	
7	-2	10	10	-4	10	-10	-

Order i	Weight w_i
1	4
2	6
3	4
4	2
5	3
6	5
7	1

$$\text{Picker capacity} = 8, \text{ Batches} = \Big\{\{3,5\},\{4,6\}\Big\}$$

3. Next largest savings pair(s): $\{1,3\},\{3,4\}$. Select $\{1,3\}$ as 3 & 4 already in a route.

   BUT $w_{1,3,5} = 11 > 8$ and $\{3,4\}$ not possible!

Heuristics Used IV

Time Savings - An Example

Order i	1	2	3	4	5	6	7
1	-						
2	X	-					
3	74	X	-				
4	64	58	74	-			
5	68	X	94	54	-		
6	X	X	X	78	68	-	
7	-2	10	10	-4	10	-10	-

Order i	Weight w_i
1	4
2	6
3	4
4	2
5	3
6	5
7	1

Picker capacity $= 8$, Batches $= \Big\{\{3,5\},\{4,6\}\Big\}$

4. Next possible pair: $\{2,7\}$

   Since $w_2 + w_7 = 7 < 8$, 2 and 7 are not in any route;

   Batches $\rightarrow$ Batches $+ \{2,7\}$

Heuristics Used V

Time Savings - An Example

Order i	1	2	3	4	5	6	7
1	-						
2	X	-					
3	74	X	-				
4	64	58	74	-			
5	68	X	94	54	-		
6	X	X	X	78	68	-	
7	-2	10	10	-4	10	-10	-

Order i	Weight w_i
1	4
2	6
3	4
4	2
5	3
6	5
7	1

$$\text{Picker capacity} = 8, \text{ Batches} = \Big\{\{3,5\},\{4,6\},\{2,7\}\Big\}$$

5. Remaining order: 1

   Since 1 cannot be added to any Batch;

   Batches $\rightarrow$ Batches $+ \{1\}$

   $\therefore$ Batches $= \{\{3,5\},\{4,6\},\{2,7\},\{1\}\}$.

Heuristics Used VI

Time Savings - An Example

Order i	1	2	3	4	5	6	7
1	-						
2	X	-					
3	74	X	-				
4	64	58	74	-			
5	68	X	94	54	-		
6	X	X	X	78	68	-	
7	-2	10	10	-4	10	-10	-

Order i	Weight w_i
1	4
2	6
3	4
4	2
5	3
6	5
7	1

$$\text{Picker capacity} = 8,\ \text{Batches} = \Big\{\{3,5\},\{4,6\},\{2,7\},\{1\}\Big\}$$

6. Now that batches of orders have been computed, use the **batch router** to compute the route of each batch.

Heuristics Used VII

Time Savings - An Example

```
savings = [ ];
for orders $o_i, o_j, o_{ij}$, $i = 1, ..., n, j = 1, ..., n, i \neq j$ do
    if $weight(i) + weight(j) \leq$ picker capacity then
        $s_{ij} = sr(o_i) + sr(o_j) - sr(o_{ij})$;
        savings$+ = [s_{ij}]$;
    end
end
Sort savings in decreasing sequence;
Batches = [ ];
for $s_{ij}$ in savings do
    if $i$ and $j \notin$ Batches then
        Batches $\leftarrow$ Batches $+ (i, j)$;
    end
    if $i \in$ Batches and $j \notin$ Batches and
     $weight(Batch\ i) + weight(j) \leq$ picker capacity
     then
        $\text{Batch}_i \leftarrow \text{Batch}_i + j$;
    end
    if $i \notin$ Batches and $j \in$ Batches and
     $weight(i) + weight(Batch\ j) \leq$ picker capacity
     then
        $\text{Batch}_i \leftarrow \text{Batch}_j + i$;
    end
end
Routes = [ ];
for Batch in Batches do
    Routes $\leftarrow br(Batch)$;
end
```

Outline

1. The Project (Recap)
   - Objectives
2. Literature Review & The Working Model
   - Preliminaries
   - The Order Batching Problem
   - The Picker Routing Problem
   - The Joint Order Batching and Picker Routing Problem
3. Solving the JOBPRP for Large Instances
   - Heuristics
   - Implementation
   - Experiments & Results
   - Conclusion & Future Work

Implementation

Routing Heuristics Overview

- The following were implemented in Python 3:
  - Shortest paths computed with NetworkX[21] library
  - Time Savings, S-shape and Largest Gap Heuristics
  - Preprocessing of warehouse parameters and raw order file data

[21] See documentation at: https://networkx.github.io/documentation/stable/

Implementation

Routing Heuristics - S-shape Code Snippet

```
node_sequence = [curr_pos]
for b in reduced_pick_blocks:
    last_pos = node_sequence[-1]
    closest_sa_and_path =
               self.get_closest_sa_from_pick(b, last_pos)
    closest_sa = closest_sa_and_path.closest_sa
    updated_b = self.reverse_subaisles_order(b, closest_sa)
    last_sa_in_b = updated_b[-1]
    for sa in updated_b:
        if updated_b.index(sa) % 2 == 0:
            node_sequence += list(reversed(sa.sa))
        elif updated_b.index(sa) % 2 != 0 and sa !=
                                      last_sa_in_b:
            node_sequence += sa.sa
        else:
            last_pick = sa.picks[-1]
            node_sequence += sa.sa[:last_pick.index+1]
```

Implementation

Setting Up A Workflow In Snakemake[22]

- A workflow management system described in Python based language
- To run experiments locally on our PCs
- To submit jobs to the cluster in NUS HPC

[22] See https://snakemake.readthedocs.io/en/stable/ for documentation.

Implementation

Setting Up A Workflow In Snakemake - Code Snippet

```
rule method1_nn_nn_d10ord10:
    output:
        "method1_d10ord10_nn_nn_8_1_3_1560.csv"
    shell:
        "python3 time_savings_heuristic.py "
        "-g ../data/foodmart/warehouseGenerator.pl "
        "-l ../data/foodmart/productsDB_1560_locations.txt "
        "-o ../data/foodmart/order/instances_d10_ord10.txt "
        "-csv {output} -c 320 -na 8 -nc 1 -ns 3 -np 1560 "
        "-sr nn -br nn"
```

Implementation

Submitting Jobs To NUS High Performance Computing (HPC)

- Experiments involving large warehouse instances were memory consuming
- Some instances took over a day to run with Method 2
- HPC made it possible to run multiple jobs at the same time
- HPC has large number of CPUs and memory at our disposal

Implementation

Submitting Jobs To NUS High Performance Computing (HPC) - Job Script Snippet

```
#!/bin/bash

## -P Exact_HPCTMP: Job project name
#PBS -P Exact_HPCTMP

## -q Queue_Name: which queue to sbmit the job to in HPC
## Note: parallel12 has wall time of 720 hours.
#PBS -q parallel12

## -l reserves 1 units of 1 cpus, 5GB memory for this job
#PBS -l select=1:ncpus=12:mem=5GB

## -j oe states to join output and error files together
#PBS -j oe

## -N Exact_Solving_Job_Output: set filename for standard output/error message.
#PBS -N Exact_HPCTMP_Job_Output

## Change to the working dir in the exec host
cd $PBS_O_WORKDIR;

##--- Put your exec/application commands below ---
## source /etc/profile.d/rec_modules.sh gets path where modules are installed
source /etc/profile.d/rec_modules.sh
module load python3.6.4
## permanently have MiniZinc driver on environment PATH in HPC
export PATH="$PATH:/home/svu/e0004335/minizinc/bin"
## exec_job is where snakemake inserts the command
{exec_job}
```

Outline

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Our Approach to Solving the JOBPRP - Heuristics I

Idea: Instantiate Time Savings[23] Heuristic (TSH) with 2 different routers

- **Savings router**: used to compute routing estimates in computing savings
- **Batch router**: used to route each picker after assignment of batched order

[23] M. B. M. De Koster , E.S. Van der Poort & M. Wolters, 1999. Efficient orderbatching methods in warehouses.

Our Approach to Solving the JOBPRP - Heuristics II

- Time Savings Heuristic has 2 parameters: savings router & batch router

	Method 1	**Method 2**	**Method 3**
Savings Router	S-shape, Largest Gap	S-shape, Largest Gap	Optimal
Batch Router	S-shape, Largest Gap	Optimal	Optimal

Breakdown of combinations of routers used in each method[24]

- PyConcorde[25], a Python wrap-around for the Concorde[26] TSP solver was used for optimal routing

[24] Cristiano Arbex Valle, John E. Beasley, and Alexandre Salles da Cunha, 2017. Optimally solving the joint order batching and picker routing problem.

[25] See PyConcorde documentation at: https://github.com/jvkersch/pyconcorde

[26] See Concorde TSP Solver documentation at: http://www.math.uwaterloo.ca/tsp/concorde.html

Our Approach to Solving the JOBPRP - Optimal Solving

- Used the JOBPRP ILP model in MiniZinc[27]
- In MiniZinc, we use the Coin-or Branch and Cut[28] (CBC) solver

[27] Based on the work of a previous intern. MiniZinc is a free and open-source constraint modeling language. See https://www.minizinc.org/

[28] CBC is an open-source mixed integer programming solver written in C++. See CBC documentation at: https://projects.coin-or.org/Cbc

Results & Analysis I

Definitions

Definition 2

$$\text{quality of solution} = \frac{\text{total distance without batching} - \text{objective value}}{\text{total distance without batching}}$$

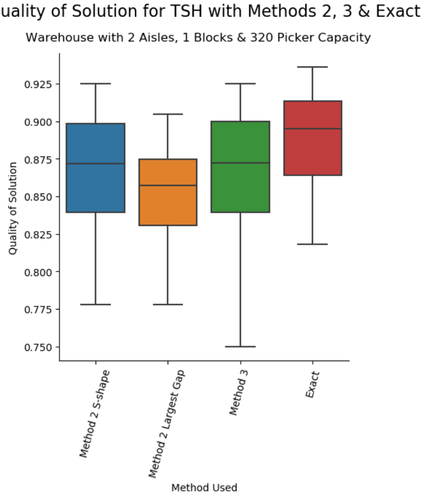
Example 3

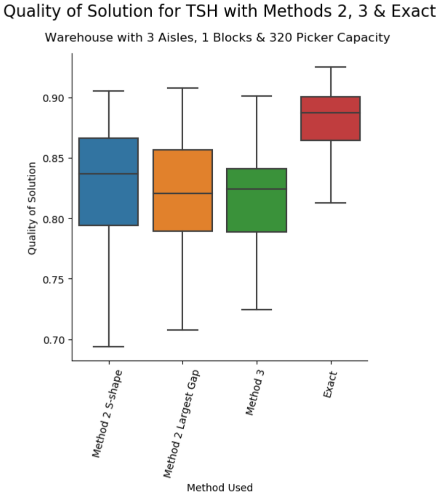
Suppose that the total distance without batching is 330, and the objective value from Method 3 is 170. Then we have

$$\text{quality of solution} = \frac{330 - 170}{330} = 0.4848$$

Results & Analysis - Exact Solving

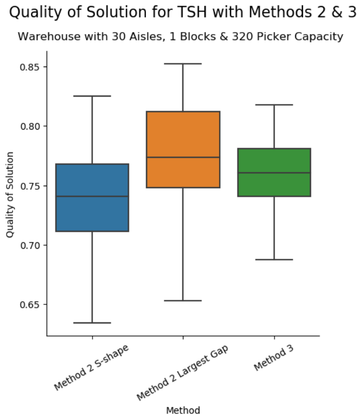
Results of Solution Quality with Methods 2 & 3, Exact Solving





Results & Analysis - Heuristic Solving I

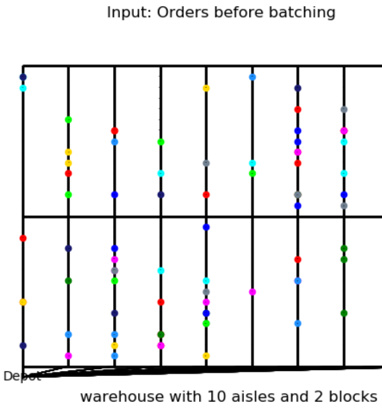
Results of Solution Quality with Methods 2 & 3



- Method 3 did not have the highest median quality of solution for most large warehouse instances

Results & Analysis - Heuristic Solving II

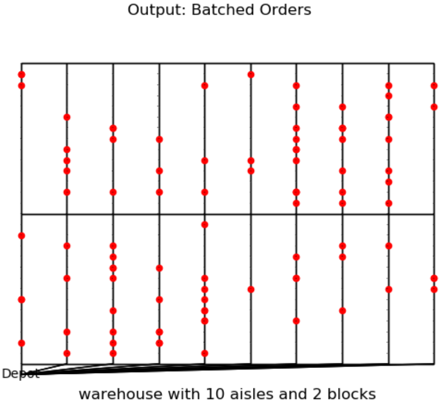
Results of Solution Quality with Methods 2 & 3


Input: Orders before batching
Depot
warehouse with 10 aisles and 2 blocks

Pick locations for each order (by color) before batching

Results & Analysis - Heuristic Solving III

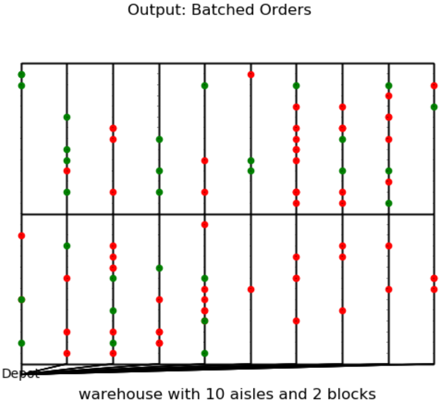
Results of Solution Quality with Methods 2 & 3



Pick locations for each order (by color) after batching with Method 2 - S-shape

Results & Analysis - Heuristic Solving IV

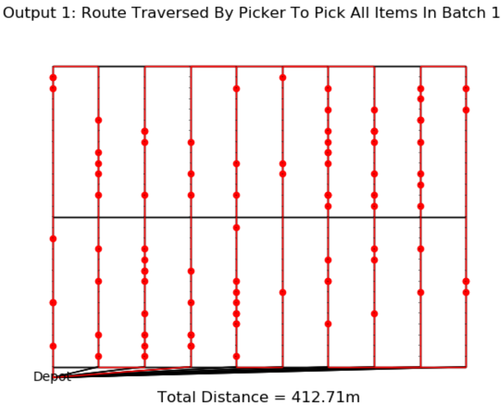
Results of Solution Quality with Methods 2 & 3



Pick locations for each order (by color) after batching with Method 3

Results & Analysis - Heuristic Solving V

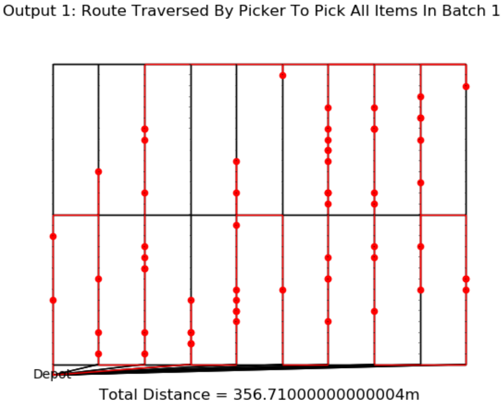
Results of Solution Quality with Methods 2 & 3


Output 1: Route Traversed By Picker To Pick All Items In Batch 1
Depot
Total Distance = 412.71m

Route with total distance for single batched order with Method 2 S-shape

Results & Analysis - Heuristic Solving VI

Results of Solution Quality with Methods 2 & 3

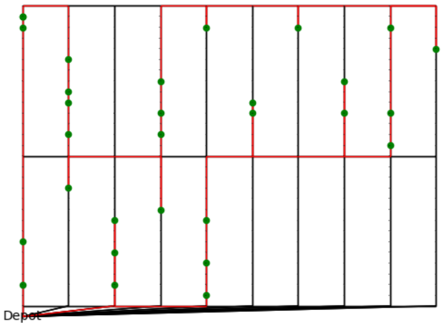

Output 1: Route Traversed By Picker To Pick All Items In Batch 1
Depot
Total Distance = 356.71000000000004m

Route with total distance for first batched order with Method 3

Results & Analysis - Heuristic Solving VII

Results of Solution Quality with Methods 2 & 3

Output 2: Route Traversed By Picker To Pick All Items In Batch 2



Total Distance = 276.81m

Route with total distance for second batched order with Method 3

Results & Analysis - Heuristic Solving VIII

Results of Solution Quality with Methods 2 & 3

- Using S-shape as savings router results in only one batched order as output
- Method 3 resulted in two batched orders as outputs
- Overall objective value is thus higher for Method 3

Results & Analysis - Heuristic Solving IX

Results of Solution Quality with Methods 2 & 3

Order *i*	1	2	3	4	5	6	7	8	9	10
1	-									
2	146.69	-								
3	156.47	156.26	-							
4	174.69	150.69	192.26	-						
5	173.09	130.69	152.47	154.69	-					
6	151.09	112.31	106.37	142.31	124.99	-				
7	100.9	86.69	100.47	100.69	86.9	72.52	-			
8	124.47	108.41	136.47	148.41	124.47	112.37	68.47	-		
9	147.09	128.69	140.47	144.69	165.09	124.71	92.9	116.47	-	
10	140.9	124.69	136.47	132.69	156.9	140.8	96.9	100.47	132.9	-

Method 2 S-shape Savings Matrix

$$\text{Batches} = \{\{3, 4, 1, 5, 9, 10, 2, 6, 8, 7\}\}$$

Results & Analysis - Heuristic Solving X

Results of Solution Quality with Methods 2 & 3

Order i	1	2	3	4	5	6	7	8	9	10
1	-									
2	96.8	-								
3	128.47	130.47	-							
4	142.42	112.8	144.37	-						
5	126.71	94.52	110.47	114.52	-					
6	112.71	92.71	92.52	104.42	100.71	-				
7	100.31	78.59	88.41	108.59	100.41	84.31	-			
8	104.47	92.47	118.47	126.47	100.47	104.37	88.41	-		
9	124.71	90.52	112.47	112.52	124.81	120.43	92.69	90.19	-	
10	126.47	110.37	122.47	120.47	132.47	128.71	92.26	112.47	124.19	-

Method 3 Savings Matrix

$$\text{Batches} = \{\{3, 4, 1, 2, 8, 7\}, \{5, 10, 6, 9\}\}$$

Results & Analysis - Heuristic Solving XI

Results of Solution Quality with Methods 2 & 3

So what happened during the batching?

Results & Analysis - Heuristic Solving XII

Results of Solution Quality with Methods 2 & 3

- Method 2 S-shape Batches $= \{\{3, 4, 1, 5, 9, 10, 2, 6, 8, 7\}\}$
- Method 3 Batches $= \{\{3, 4, 1, 2, 8, 7\}, \{5, 10, 6, 9\}\}$
- In Method 3's batching, Order 5 does not get added to the first batch $\{3, 4, 1\}$

Results & Analysis - Heuristic Solving XIII

Results of Solution Quality with Methods 2 & 3

Step 1 : $\{\{3, 4\}\}$
Step 2 : $\{\{3, 4, 1\}\}$
Step 3 : $\{\{3, 4, 1, 5\}\}$

Steps of Method 2's batching

Step 1 : $\{\{3, 4\}\}$
Step 2 : $\{\{3, 4, 1\}\}$
Step 3 : $\{\{3, 4, 1\}, \{5, 10\}\}$

Steps of Method 3's batching

Results & Analysis - Heuristic Solving XIV

Results of Solution Quality with Methods 2 & 3

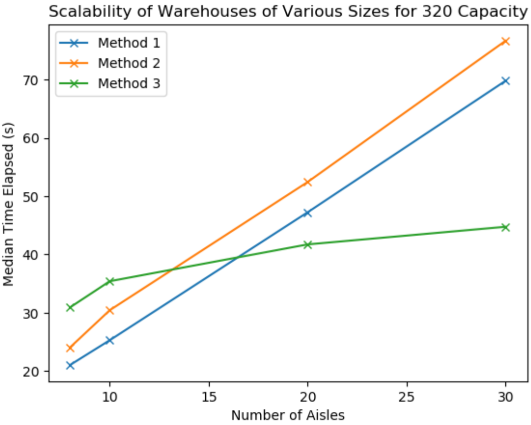
Order *i*	1	2	3	4	5	6	7	8	9	10
1	-									
2	96.8	-								
3	128.47	130.47	-							
4	142.42	112.8	144.37	-						
5	126.71	94.52	110.47	114.52	-					
6	112.71	92.71	92.52	104.42	100.71	-				
7	100.31	78.59	88.41	108.59	100.41	84.31	-			
8	104.47	92.47	118.47	126.47	100.47	104.37	88.41	-		
9	124.71	90.52	112.47	112.52	124.81	120.43	92.69	90.19	-	
10	126.47	110.37	122.47	120.47	132.47	128.71	92.26	112.47	124.19	-

Method 3 Savings Matrix

- For Method 3, order pair $(5, 10)$ had a higher savings than $(1, 5)$ in the third step
  - $s_{5,10} = 132.47 \geq 126.71 = s_{1,5}$

Results & Analysis - Heuristic Solving XV

Results of Solution Quality with Methods 2 & 3

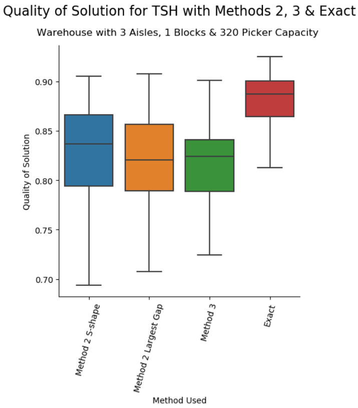


- Method 3 had the lowest median run-time than Methods 1 & 2 for 20 and 30 aisles

Outline

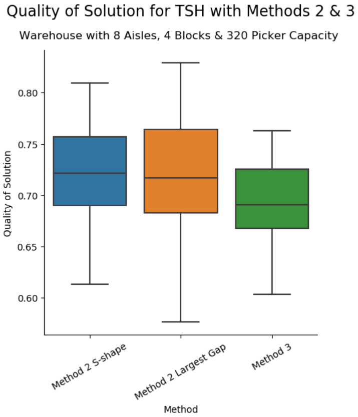
1. The Project (Recap)
   - Objectives
2. Literature Review & The Working Model
   - Preliminaries
   - The Order Batching Problem
   - The Picker Routing Problem
   - The Joint Order Batching and Picker Routing Problem
3. Solving the JOBPRP for Large Instances
   - Heuristics
   - Implementation
   - Experiments & Results
   - Conclusion & Future Work

Conclusion I



- We investigated the trade-offs of using heuristics in-place of exact solving

Conclusion II

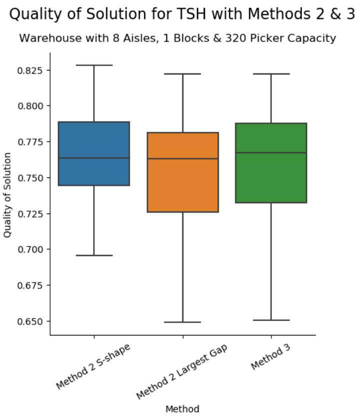


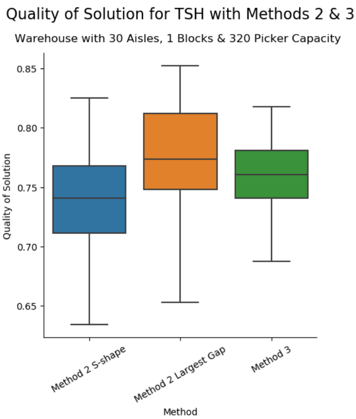
- Routing heuristics do indeed give a better quality of solution than optimal routing in most cases

Conclusion III

- ILP formulation takes a long time to solve, heuristics are faster

Conclusion IV





- Different methods and heuristics work better for certain warehouses

Future Work

- Improve the run-times for Method 2
- Batching heuristic using optimal routes of all items to cluster orders